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Absorption versus Direct Costing: The Relevance of Opportunity Costs in the Management of Congested Stochastic Production Systems

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Absorption versus Direct Costing: The Relevance of Opportunity Costs in the Management of Congested Stochastic Production Systems [1]

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Abstract

This paper investigates the performance of absorption versus direct costing procedures. The setting for our analysis is a product admittance problem that takes place in a stochastic and dynamic production system. We consider the extent to which absorption costing based accounting calculations provide good proxies for hard to observe opportunity costs. We show that the existence of opportunity costs cannot always be used as a defense of absorption costing. In order to guide the comparative ranking of costing procedures, we show the existence of an "open admittance" condition on the parameters of the problem that ensures that absorption costing always out performs direct costing. We conclude by discussing the implications our theory has for the empirical analysis of the absorption costing versus direct costing debate.

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1 Introduction

In this research, we focus on the enduring question of whether to base operating decisions made in an uncertain environment on either absorption or direct costing procedures. Zimmerman (1979) has proposed that "... cost allocation can act as a lump-sum tax which reduces the manager's consumption of perquisites and that cost allocations can serve as useful proxy variables for certain difficult to observe costs." In effect, he argues that cost allocation can perform the dual role of alleviating both agency cost problems and problems associated with the determination of opportunity costs. In this paper, we systematically investigate the extent to which one can defend absorption costing (cost allocation) on the basis that it is a good proxy for opportunity costs.

Our analysis takes place within the context of a simple manufacturing environment. Two products, endowed with differential contribution margins, but identical processing requirements, are produced in a manufacturing facility with limited capacity. Orders for the products arrive randomly over time and, upon their arrival, may be either accepted or rejected. Processing times of accepted orders are random.

All jobs waiting for processing incur a holding cost per unit time during the time they remain in the facility. The decision problem is to determine whether an arriving order should be accepted or rejected so that the total expected net profit (contribution margin less holding cost) generated over a prescribed length of time is maximized.

The problem is complicated by the fact that the number of jobs in the system fluctuates randomly over time so that the holding cost generated by an admitted order is difficult to measure. Furthermore, the acceptance of orders for the product with the low-contribution margin reduces the processing capacity available for any high-contribution margin orders that may arrive in the future. The opportunity costs associated with the admittance of low-revenue orders are also difficult to measure—they depend upon the status of the facility, a characteristic that changes randomly over time.

From a practical point-of-view, how would a cost accounting system determine which orders should be admitted? The answer to this question centers on the measurement of relevant costs. In practice, two heuristic rules are commonly employed to control the admittance to stochastic service systems such as the one we analyze here. (See Dickhaut and Lere

(1983) for a discussion of heuristics involving absorption costing used in practice.) These "rules-of-thumb" are motivated by ideas grounded in prescriptions for dealing with fixed costs, capacity, and congestion in cost accounting systems. The *direct costing (DC) rule* admits any order whose contribution margin exceeds the expected holding cost associated with the order if it is admitted. This rule views the cost of providing the productive capacity as sunk, and thus ignores it. Furthermore, this rule ignores the opportunity cost generated by the admittance of an order.

The second rule-of-thumb observed in practice seeks to account for opportunity costs generated by an order that is admitted to the facility. It does this by allocating to all admitted orders some portion of the fixed cost of providing the capacity of the facility. The *absorption (or full) costing (AC) rule* admits an order if its contribution margin exceeds the sum of its expected holding cost and the allocated cost of providing the productive capacity.

Is one of these rules universally better? If not, under what conditions will one be better than the other? In principle, we know there is an *optimal* admittance rule that correctly balances contribution margins and total economic costs. The DC rule can only underestimate the relevant costs associated with an order and hence can only over-admit relative to an optimal admittance policy. If allocated costs are sufficiently large, the AC rule may over-estimate opportunity costs and, as a result, under-admit relative to the optimal rule. Unlike the DC rule, the AC rule can both under- and over-admit. A priori, it is not clear which rule is better—is it better to both under- and over-admit a little or to not under-admit at all but over-admit a great deal? Are there conditions under which we would expect the absorption costing rule to generate higher expected rewards than the direct costing rule?

To address these and other questions regarding the measurement of relevant economic costs in a stochastic setting, we formulate the problem of determining which orders should be admitted to the production facility as a Markov decision process (MDP) in which we maximize expected total contribution less holding cost generated over a given length of time. The MDP is an optimization procedure that will yield as its solution an *optimal* admittance policy that prescribes whether or not an arriving order should be admitted based upon the current number of jobs that are already in the facility and the length of time that remains in the planning horizon. When a new order arrives at the facility, the MDP implicitly compares the contribution margin that the order will generate upon the completion of its processing

with the expected total cost that the order will incur while it is in the system. This cost is the sum of the (direct) holding cost generated by the order *plus the actual opportunity cost that that order will impose on the system.*

The focus of this paper is not on the use of MDP's to optimally control the admittance of orders to a production facility. Rather our interest centers on what the MDP allows us to say regarding the efficacy of cost accounting based admittance rules in a stochastic operating environment. A principle contribution of this paper is the derivation of conditions on the observable parameters of the problem that ensure that product acceptance rules that are based on absorption costs always dominate direct costing based rules. In particular, we show that if allocated costs are not "too high", we can unambiguously declare that absorption costing results in higher expected profits than direct costing for all possible in-process inventory levels. Furthermore, we explicitly define the notion of "high" allocated costs in terms of the parameters of the problem. We therefore provide greater rigor to the traditional defenses of cost allocation.

Banker, Datar, and Kekre (1988, hereafter BDK) contribute to our understanding of the nature of relevant (opportunity) costs by showing why it is important to include inventory carrying costs (holding costs) in cost analysis. However, they are primarily concerned with issues regarding the *design specification* of a manufacturing system in which the decision whether to accept a new product or provide investment in plant to reduce setup time is made once, at the beginning of a planning period. For the product admittance problem that we consider, the design specification is set. We are concerned with the determination of *operational* (micro-level) admittance rules that control the operation of the facility over time.

Similarly, Miller and Buckman (1987) consider issues regarding the *design specification* of a stochastic production system that includes the choice of arrival and service rates. Their research strategy of relating allocated fixed costs to the optimal opportunity costs arising in a queueing system model of the production process is commensurate with ours. However, as with BDK, their focus is on design specification issues. In particular, they are concerned with the determination of the type of production facility that will be employed and not on the operation of a facility once it has been specified.

The paper is organized as follows. Section 2 contains a discussion of opportunity costs

and related literature. We argue that an expansive view of opportunity cost issues that arise in managerial decision making processes takes place within a stochastic, dynamic operating environment. In such an environment, actual opportunity costs depend upon the ever-changing status of the system. An MDP model of the product admittance problem is formulated in Section 3. We use this model to identify optimal product admittance rules that into account the actual opportunity costs associated with operating in a congested stochastic production system. A numerical example is used to illustrate just how complex the optimal opportunity cost measures may become. In Section 4, we consider the product acceptance rules derived from the conventional cost accounting techniques of direct costing and absorption costing. Here we derive an expression that is a lower bound on the expected holding cost that an order incurs if it admitted to the system. A numerical example illustrates how these rules compare to the optimal admittance policy and show that in general, one rule does not always dominate for all possible values of the parameters of the problem.

To this point, therefore, we are left with no clear guidelines of which costing procedure to support. In Section 5, we overcome this problem by developing an "open admittance" condition on the parameters of the problem that allow us to unambiguously recommend the adoption of absorption costing rather than direct costing procedures. We present a clear intuition for our results and discuss how the condition could be useful in practice. We then briefly discuss empirical implications of the open admittance condition. Concluding remarks are contained in Section 6.

2 The Nature and Measurement of Opportunity Costs

More than forty years ago, Devine (1950, pp. 389) raised the issue of the need for cost accountants to explicitly consider the nature and significance of congestion costs resulting from operational capacity constraints. He recounts how "... businessmen ... have ... favored the cost accountant's total unit cost ..." concept, now often referred to as absorption costing, in which an allocated share of sunk costs is charged against products when making order acceptance decisions. He defends this practice by arguing that, if "... a firm is operating at full capacity ..., the usual approach that utilizes the contribution of the selling price over variable costs must be modified drastically before it can be applied with benefit." He supports

absorption costing practices on the grounds that "... the distribution of fixed overhead to jobs or products is normally on a time basis, and the relative total fixed overhead charges to jobs do therefore measure more or less imperfectly the relative usage of the firm's scarce factor of production."

Two observations flow naturally from these statements. First, direct costing procedures are seen as imperfect, in the sense that no incorporation of opportunity costs arising from capacity (congestion) constraints are incorporated. Second, he does not present an unequivocal recommendation for the use of absorption costing. This arises because his defense of absorption costing depends on whether or not a firm is operating at full capacity. Devine's rationale for absorption costing does not apply to a firm operating consistently below capacity, as is often recommended by manufacturing ideologies such as just-in-time. It is *as if* a firm needs to have a bell that rings when capacity is reached and then, instantaneously, switches from direct costing to absorption costing. This latter remark is not intended in any way to trivialize Devine's argument, but instead is used to motivate a critical consideration of his implicit model of the production environment. Our fundamental criticism of Devine's rationale centers on the concept of opportunity (congestion) cost that he uses. Implicit in Devine's discussion of opportunity cost is the belief that it is sufficient to characterize the production environment as deterministic and static. In such an environment, a "... specific resource is not considered to be scarce ... unless it is optimal to exhaust the capacity of the resource completely" (Knudsen 1972, pg. 526). The result is a "bang-bang" notion of opportunity cost (shadow pricing) in which opportunity costs are characterized by fundamental discontinuities that jump from zero to non-zero values when full capacity is reached.

If we are to develop an *expansive theory* of absorption costing that is based upon the need to incorporate opportunity costs, we must do so in a production environment wherein opportunity costs depend on *any* operating load of the system, not just a load that corresponds to full capacity. Given this specific intent, it is therefore natural to consider the production environment as a stochastic process. We specify order arrival rates and manufacturing processing rates so that the expected proportion of capacity usage is less than 100%. Even though there is less than 100% utilization of the facilities, opportunity costs are relevant. At any point in time, there is a positive probability that the facility is operating at full capacity in the sense that the job currently being processed or the jobs that are awaiting processing

may not be completed within a prescribed length of time. Furthermore, the decision whether or not to admit a product to the production facility will positively affect the opportunity cost of *future* product orders. Neither of these effects can arise in a deterministic and static production setting. (See Knudsen (1972) and BDK for more details.)

The problem of deciding which products to admit to a congested production system is a special case of the general problem of optimally controlling the admission of customers to a (congested) queueing system. In the following section, we develop a queueing-control model that specifies optimal expected profits in terms of the number of jobs in the facility. We then present a numerical example to demonstrate the application of the model; this example also illustrates a number of desirable features of the model.

We have thus achieved our initial goal of establishing a model of the production environment in which decisions regarding product admittance give rise to a more profound notion of (mutable) opportunity cost. Moreover, our analytical development of the optimality equations of the MDP allow us to discuss in an unambiguous fashion the *exact* measurement of opportunity cost. This arises because the use of the optimal control procedure for admission to a queue is equivalent to charging an optimal toll for entrants to a congested system. This equivalence was first informally proposed by Leeman (1964, 1965) and Saaty (1965), and was addressed analytically in Naor (1969). Using our MDP formulation, we know that the *optimal toll* for the queue admittance control problem represents the *opportunity cost* that we wish to identify. (See Lippman and Stidham (1977) for a related discussion of optimal tolls in congested queueing systems.)

As our numerical example makes clear, the complex nature of implied opportunity cost makes it is extremely difficult to compute, and therefore to use, in practice. Clearly this measurability issue must be addressed in order for us to be able to develop costing procedures that are readily implementable. With Zimmerman's initial assertion in mind, we consider the extent to which absorption costing procedures proxy opportunity costs that have been identified by the optimal control formulation.

3 Optimal Admittance Rules

We now develop a queueing theory model of the production process in which we identify the actual opportunity cost of acceptance of arriving product orders. For simplicity, we model the manufacturing facility as an M/M/1/I+J+1 queueing system (i.e., Possion arrivals, exponential service times, one server, and a system capacity of $I + J + 1$ jobs) with two customer classes (product types). The single-server assumption allows us to concentrate on a congested production facility that processes one job at a time. Arrivals (product orders) occur at a rate of $\lambda > 0$ per unit time so that times between arrivals are exponential random variables with mean $1/\lambda$. Each arrival belongs to either the high-revenue class with probability p or the low-revenue class with probability $1 - p$. High (low) revenue jobs have a contribution margin of r_1 (r_0 with $r_1 > r_0$) upon completion; these margins are per unit and net of unit variable cost. For expositional purposes, we refer to jobs whose contribution margins are r_1 and r_0 as high and low revenue jobs, respectively.

The server operates at a rate of μ jobs per unit time, where $\mu > \lambda$. Under the M/M/1 assumption, processing times are exponential random variables with mean $1/\mu$. BDK explain in detail why it is necessary to include the "incremental carrying or holding costs of inventories from longer queues" in cost analyses. In order to ensure that it will not be desirable to always admit every order if there is room, we assume that when an order is admitted to the system, it incurs a holding cost of \$1 per unit time during the entire time the job is in the facility. At each arrival epoch, the firm observes the type (contribution) of the order and may either accept or reject that order based upon the number of orders of each type that are currently in the system. When an order is accepted it is referred to as a job.

In our production environment, opportunity costs are generated whenever a low-revenue product is accepted. Some of the finite capacity of the manufacturing facility is used up to process the low-revenue jobs, leaving less capacity to process high-revenue jobs that may arrive later. The decision-maker is thus faced with the problem of deciding whether or not to accept the low-revenue job. If a low-revenue job is accepted, there may not be enough time in the budget period to process a high-revenue order that may become available in the future. If a low-revenue job is not accepted, there is positive probability that there will be no high-revenue orders in the future.

Before we formally describe the operating environment by defining the states of the decision process, we need to specify how the budget period and queue disciplines are set. The queue discipline we employ is a modified FIFO scheme. Within each of the two job classes, jobs are processed on a first-in-first-out basis. However, the choice of the job class to process next is stochastic. Upon a service completion, the server selects a high-revenue job from the queue of accepted high-revenue jobs with probability $\gamma(i,j)$ and from the queue of low-revenue jobs with probability $1-\gamma(i,j)$ when there are i and j high and low-revenue jobs in the queue, respectively. We assume that the scheduling rule $\gamma(i,j)$ has the following simple form:

$$\gamma(i,j) = \begin{cases} \beta & \text{if } i>0, \quad j>0 \\ 1 & \text{if } i>0, \quad j=0 \\ 0 & \text{if } i=0, \quad j>0 \end{cases}$$

for $0 \leq \beta \leq 1$. Thus, if there are both types of jobs waiting to be processed, β is the probability that a high-revenue job is selected.

We assume that the decision process proceeds for T units of time, an exogenous variable that we refer to as the *budget period.* This corresponds to the length of time the server is left to operate without re-evaluating its capacity. Thus, it is natural to view the budget period as the length of time between periodic evaluation of the design considerations that arise in Miller and Buckman and BDK. In queueing control models, decisions can be made only at arrival or service completion epochs. The number of "decision periods", therefore, corresponds to the number of arrivals (orders) and departures (service completions) that occur during T units of time. In our model, the time until either the next arrival or the next departure is an exponential random variable whose mean is $1/(\lambda+\mu)$. Let $N(\lambda,\mu)$ denote the expected number of decision periods that constitute a single budget period. Thus, $N(\lambda,\mu) = (\lambda+\mu)T$. Without loss of generality, we scale time so that $T=1$ and will, for simplicity, refer to $N(\lambda,\mu)$ as the budget period. We assume that the revenue associated with any job that has been accepted but not processed completely before the end of the budget period is foregone.

We develop our formulation of the operating environment via the following state space characterization. Transitions between states of the decision process occur as the result of the arrival of an order or the completion of the processing of a job. If the transition is due to an arriving order, let (i,j,r_s,r_a) denote the state, where i and j are the number of high-revenue

and low-revenue orders, respectively, that are already in the queue. In addition, r_s and r_a are the revenues associated with the job currently in service and with the arriving order, respectively. If the transition is due to a service completion, the state is denoted simply as (i, j, r_s), where r_s is the revenue of the new job just selected for service and $i + j$ are the number of remaining jobs waiting for service in the queue.

We assume that there is limited storage area for both types of jobs. In particular, there can be no more than I and J high-revenue and low-revenue jobs, respectively, in the queue at any time.

3.1 A Continuous-Time MDP

We now develop notation that is used to define a model that will yield optimal admittance policies. The model compares the net benefits of not admitting a product that arrives with the benefits of admittance. Let r_a be the contribution margin of an arriving order and r_s denote the contribution margin from the order that is currently in service; if no job is currently in service, $r_s = 0$. The desirability of admittance also depends on the congestion in the system, which is determined by the current values of i, j, and r_s. To determine the net benefits from *admitting* an arriving order, let $V_t(i, j, r_s, r_a)$ denote the optimal expected discounted total profit generated from the beginning of period t through the end of the budget period, where $t = 1, \ldots, N(\lambda, \mu)$ when the state at the beginning of period t is (i, j, r_s, r_a). Let $U_t(i, j, r_s)$ denote the optimal total expected discounted profits earned from period t onward when the state is (i, j, r_s). Let

$$\mu(r_s) = \begin{cases} \mu & \text{if } r_s > 0 \\ 0 & \text{otherwise} \end{cases}$$

denote the service rate as a function of r_s, the revenue of the job in service. We assume that the total holding cost rate per unit time incurred by the system when there are i (j) high- (low-) revenue jobs in the queue and is of the form:

$$h(i, j, r_s) = \begin{cases} i + j + 1 & \text{if } r_s > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

We present the uniformized version of a continuous-time Markov decision chain, in which the distribution of holding times in each state is independent of the action taken in that state. Let $\Lambda = \lambda + \mu$ denote the parameter of the exponential time that the process spends

in any state. Using this notation, the budget period is $N(\lambda, \mu) = \Lambda$. Let $\alpha \geq 0$ denote the continuous discounting rate, so that the present value of $\$x$ received τ time units from now is $xe^{-\alpha\tau}$.

3.2 The Optimality Equations

The $V_t(\cdot,\cdot,\cdot,\cdot)$ and $U_t(\cdot,\cdot,\cdot)$ functions satisfy the following dynamic programming equations: If $r_s > 0$,

$$V_t(i,j,r_s,r_1) = \max\{U_t(i+1,j,r_s) - U_t(i,j,r_s), 0\} + U_t(i,j,r_s) \tag{2}$$

for $0 \leq i \leq I-1$ and $j \leq J$ and

$$V_t(i,j,r_s,r_0) = \max\{U_t(i,j+1,r_s) - U_t(i,j,r_s), 0\} + U_t(i,j,r_s) \tag{3}$$

for $0 \leq j \leq J-1$, $i \leq I$, and $1 \leq t \leq N(\mu)$, where

$$\begin{aligned} U_t(i,j,r_s) \;=\; & \{-h(i,j,r_s) + \lambda[pV_{t+1}(i,j,r_s,r_1) + (1-p)V_{t+1}(i,j,r_s,r_0)] \\ & +\mu(r_s)[r_s + \gamma(i,j)U_{t+1}(i-1,j,r_1) \\ & +[1-\gamma(i,j)]U_{t+1}(i,j-1,r_0)]\}/(\alpha+\Lambda), \end{aligned} \tag{4}$$

if $i + j > 0$.

Also

$$V_t(I,j,r_s,r_1) = U_t(I,j,r_s)$$

for all j and

$$V_t(i,J,r_s,r_0) = U_t(i,J,r_0)$$

for all i, since a new job cannot be admitted if the input buffer is full for that job type.

If $r_s = 0$, then $i + j = 0$ and

$$V_t(0,0,0,r_a) = \max\{U_t(0,0,r_a) - U_t(0,0,0), 0\} + U_t(0,0,0) \tag{5}$$

for all r_a. Also, $U_t(-1,\cdot,\cdot) = U_t(\cdot,-1,\cdot) = 0$ for all t, $U_{N(\lambda,\mu)+1}(\cdot,\cdot,\cdot) = 0$, and

$$\begin{aligned} U_t(0,0,r_s) \;=\; & \{-h(0,0,r_s) + \lambda[pV_{t+1}(0,0,r_s,r_1) \\ & +(1-p)V_{t+1}(0,0,r_s,r_0)] \\ & +\mu[r_s + U_{t|1}(0,0,0)]\}/(\alpha+\Lambda) \end{aligned} \tag{6}$$

for $r_s > 0$ and

$$U_t(0,0,0) = \{\lambda[pV_{t+1}(0,0,0,r_1) + (1-p)V_{t+1}(0,0,0,r_0)] + \mu U_{t+1}(0,0,0)]\}/(\alpha+\Lambda) \quad (7)$$

for $1 \leq t \leq N(\mu)$.

We begin the interpretation of these equations with (4). The first term $(-h(i,j,r_s)/(\alpha+\Lambda))$ represents the discounted total expected holding cost generated during the exponential amount of time the system is in state (i,j,r_s). If the period ends as the result of an arrival, which occurs with probability λ/Λ, the new arrival is a high- (low-) revenue order with probability p ($[1-p]$). The maximum expected total discounted profit that can be generated from period $t+1$ through period $N(\lambda,\mu)$ is $V_{t+1}(i,j,r_s,r_a)$, where r_a is the revenue associated with the arrival. The term in (4) that is multiplied by λ is the maximum total expected profit that can be generated if period t ends as the result of an arrival and the decision process continues optimally. Similarly, the term that is multiplied by $\mu(r_s)$ is the maximum expected total discounted profit that can be earned from period $t+1$ onward if period t terminates as a result of a service completion. The revenue r_s associated with the job in service during period t is received, the state of the decision process moves from (i,j,r_s) to either $(i-1,j,r_1)$ or $(i,j-1,r_0)$ with probability $\gamma(i,j)$ and $1-\gamma(i,j)$, respectively.

The expression in (2) quantifies the options that are available when a high-revenue order is being considered for admittance. If the order is admitted, it joins the high-revenue job queue, which increases in length from i to $i+1$. If the order is declined, the high-revenue job queue remains at its current length of i. In either case, the decision process continues in an optimal manner.

The expression for $V_t(0,0,0,r_a)$ in (5) reflects our assumption that an order that is admitted to an empty system begins processing immediately.

Finally, the expression for $U_t(0,0,0)$ in (7) indicates what can happen when the facility is empty: there may an arrival of an order (either high or low revenue) or there may be a "fictitious" service completion — the job currently in service remains in service for another time period. The latter event is part of the uniformization process; see, e.g., Lippman (1975).

We now present a numerical example that illustrates the opportunity costs generated by accepting low-revenue orders.

3.3 Numerical Example

From the dynamic programming optimality equations, we know that the optimal expected payoff when considering the acceptance of a low-revenue order is given by the expression in (3): a low-revenue job should be admitted if, and only if, the optimal expected discounted total profit that can be earned over the remainder of the budget period is greater in state $(i, j+1, r_s)$ than it is in state (i, j, r_s). A particularly desirable feature of this formulation is that $U_n(i, j+1, r_s) - U_n(i, j, r_s)$ is the explicit measure for the opportunity cost, since it measures the net impact of admitting one more low-revenue order to the production system. Whenever the opportunity cost associated with the acceptance of a low-revenue order is negative, it is not optimal to admit that order. To illustrate this point, we use the MDP to determine the numerical values of the opportunity cost that depend on the state of the system for the set of parameter values in Table 1.

Place Table 1 here.

The optimal expected profit and opportunity cost for several states are given in Table 2 when there are no high-revenue jobs in the queue but there is a high-revenue job in service in the first period of a problem whose budget period consists of 41 periods.

Place Table 2 here.

From the information in Table 2, we see that it is optimal to admit at most four low-revenue jobs to this system when there are no high-revenue jobs in the queue and there is a high-revenue job at the server. We describe this optimal admittance policy as a "$j^*_{\text{MDP}} = 4$ policy", remembering that the critical number of jobs depends on the specific values of t, i, and r_s. For $i = 0$ and $r_s = 0.3$, the complete j^*_{MDP} policy for all periods is given in Table 3.

Place Table 3 here.

For realistic production settings, the computational complexity of the MDP is considerable, since the i, j, and r_s values vary from period to period. Therefore, in the next section, we consider two heuristic costing rules often used in practice to determine admittance policies. Given our identification of optimal policies in this section, we can now be unambiguous in our appraisal of these (possibly) non-optimal procedures.

4 Direct and Absorption Costing Based Admittance Rules

In this section, we explicitly develop admittance rules based upon direct and absorption costing assumptions. As explained in BDK, costing procedures developed to operate in congested and stochastic production environments need to include holding cost in the analysis. The continuous-time MDP of the previous section did so optimally, by explicitly incorporating holding costs within the dynamic programming formulation (see equations (4) and (6)). Thus, the direct and absorption costing procedures must also consider holding costs. We cannot, however, use the identical holding costs implicitly used by the MDP. The rationale for considering heuristic cost accounting procedures was that the optimal analysis could not be readily implemented in practice. The actual holding cost associated with an admitted order is a complex (random) function that depends in part on the rule used to admit orders to the system. Thus, we identify approximations for the holding costs involved that are readily measurable and hence, implementable. We do this in Section 4.1. A desirable feature of our analysis is that we can develop upper and lower bounds on the expected holding costs and, hence, can assess the accuracy of these bounds.

4.1 Expected Holding Times

First, we determine an upper bound on the expected amount of time a low-revenue job spends in the system when there are i and j other high- and low-revenue jobs waiting for processing. Assume that the high-revenue input queue never empties. Under this assumption, there is always a probability of $(1-\beta)/\mu$ that a low-revenue job is selected for processing. The amount of time a low-revenue jobs remains in the system is overstated under this assumption. (If the high-revenue input queue is empty, then, with probability 1, the a low-revenue job is selected for processing.) Since jobs are processed in a FIFO manner, we want to determine the number of periods that must elapse before we have exactly $j+1$ low-revenue job completions.

Let N_{j+1} be a random variable that denotes the number of decision periods that elapse before a job that is admitted when there are j jobs already in the low-revenue input queue. Decision periods are independent exponential random variables with mean $1/\Lambda$. In light of the discussion above, N_{j+1} is a negative binomial random variable with parameters $(j+$

$1, (1-\beta)\mu/\Lambda)$, so that

$$\mathrm{P}\{N_{j+1} = m\} = \binom{m-1}{j} \left[\frac{(1-\beta)\mu}{\Lambda}\right]^{j+1} \left[\frac{\lambda+\beta\mu}{\Lambda}\right]^{m-j-1}$$

for $m \geq j+1$. Therefore,

$$E[N_{j+1}] = \frac{(j+1)\Lambda}{(1-\beta)\mu}.$$

Let $\overline{H}_j$ be a random variable that denotes the holding time (the total time spent in the system) of our new arrival. Under the assumption that the high-revenue queue never empties, $\overline{H}_j$ is an upper bound on the actual time this job remains is the system. Using Wald's equation, an upper bound on the expected amount of time until our new arrival leaves the processing station is

$$\mathrm{E}[\overline{H}_j] = E[N_{j+1}]/\Lambda = \frac{j+1}{(1-\beta)\mu}.$$

We obtain a lower bound on the expected holding time, denoted by $\underline{H}_{j+1}$, by assuming that every job completion corresponds to the completion of a low-revenue job. Under this assumption, the parameters of N_{j+1} are $(j+1, \mu/\Lambda)$, so that

$$\mathrm{E}[\underline{H}_{j+1}] = (j+1)/\mu.$$

In the remainder of this section, it will be convenient to refer to the number of periods *remaining* in the budget period rather than the number of the period. If the period number is t, then $n = N(\lambda, \mu) - t + 1$ denotes the number of periods remaining (including period t).

4.2 Direct Costing Admittance Rules

In our stochastic environment, a direct costing rule admits a job if its revenue exceeds the expected holding cost generated by that job. (Recall that revenue is defined net of variable costs and hence is equivalent to the contribution margin.)

If the state of the process is (i, j, r_s, r_0) at the beginning of period t, we do not know for certain how many jobs will be processed over the remaining n periods, where $n = N(\lambda, \mu) - t + 1$. Furthermore, we do not know for certain how many low-revenue jobs will be processed. In addition, revenue is not received until after the processing of a job is complete. The direct costing admittance rule we develop in this section determines the difference between the

expected revenue (with respect to the probability that the job is completed) and the expected holding costs (with respect to both the probability that low-revenue jobs are processed and the probability that the processing is completed before the end of the budget period). Note that with inclusion of holding costs in the formulation, capacity considerations may force management to reject (under direct costing) an arriving product order with a positive contribution margin. Thus, management is concerned with satisfaction of both the costing admittance rule and capacity constraints.

Suppose that the first parameter of N_{j+1} is $j+1$. If $N_{j+1} = m$, the average time spent in the system by our new arrival (and hence its expected holding cost) is m/Λ. Therefore, the expected contribution margin net of holding costs is $r_0 - m/\Lambda$. The probability that this contribution is earned is $\mathrm{P}\{N_{j+1} \leq n\}$, when there are n periods remaining in the budget period. If $N_{j+1} > n$, r_0 is not received and the expected holding cost is n/Λ. Therefore, the expected contribution margin of the new arrival whose revenue is r_a, if it is admitted and if we use the lower bound estimate for the holding costs, is

$$\begin{aligned} D(n,j,r_a) &= \sum_{m=j+1}^{n} (r_a - m/\Lambda) P\{N_{j+1} = m\} + \sum_{m=n+1}^{\infty} -(n/\Lambda) P\{N_{j+1} = m\} \\ &= \sum_{m=j+1}^{n} \left[r_a + \left(\frac{n-m}{\Lambda} \right) \right] \mathrm{P}\{N_{j+1} = m\} - \frac{n}{\Lambda}. \end{aligned} \tag{8}$$

The *Direct Cost Admittance Rule* is:

> Admit a low-revenue job if, and only if, $D(n,j,r_0) \geq 0$, $i+j+1 \leq n$, and $j+1 \leq J$. Admit a high-revenue job if, and only if, $D(n,i,r_1) \geq 0$, $i+1 \leq n$, and $i+1 \leq I$.

Since we are using the lower bound on the holding costs, we know that this version of the direct costing rule cannot under-admit arrivals relative to the optimal admittance policy. This is intuitively appealing since direct costing makes no allowances for the opportunity cost and hence tends to over-admit (and thus overly congest) the production system as suggested by Devine. Note, however, that at this stage, the fact that direct costing admittance rules may not be optimal in no way establishes the relative desirability of absorption costing rules. *Appraisal of relative desirability can only justifiably be made with reference to the optimal admittance policy.*

4.3 Absorption Costing Admittance Rules

The admittance rules under absorption costing are similar to those under direct costing. However, an additional cost is considered, which may proxy the opportunity costs of congestion that are computed exactly in the MDP. Let $K(\mu)$ denote the (fixed) capital cost associated with the provision of the (manufacturing) server that operates at rate μ units per unit time during the current budget period. Each unit processed during the budget period is allocated a share of this sunk cost equal to $K(\mu)/\mu$.

As before, suppose that the state of the decision process is (i, j, r_s, r_a) and n periods remain in the budget period. Let

$$A(n, i, r_a) = \sum_{m=i+1}^{n} \left[r_a - K(\mu)/\mu + \left(\frac{n-m}{\Lambda} \right) \right] P\{N_{i+1} = m\} - \frac{n}{\Lambda}. \tag{9}$$

The *Absorption Cost Admittance Rule* is:

> Admit a low-revenue order if, and only if, $A(n, j, r_0) \geq 0, i + j + 1 \leq n$, and $j + 1 \leq J$. Admit a high-revenue order if, and only if, $A(n, i, r_1) \geq 0, i + 1 \leq n$, and $i + 1 \leq I$.

We will now illustrate the application of these costing rules in the stochastic production environment.

4.4 Numerical Example Revisited

We adopt the same parameter values given in Table 1 and additionally assume that the sunk cost is 2 cost units (i.e., $K(\mu) = 2$). The admittance policy for the MDP is clearly unchanged. The *form* of the AC and DC rules are similar to the MDP rule: for each n, i, and r_s value, each rule admits a low-reward order if, only if, the number of low-reward jobs awaiting processing is smaller than some critical number. The critical numbers defining the AC and DC admittance rules are denoted as j^*_{AC} and j^*_{DC}, respectively. The two rules for the accounting-based costing procedures are given in Tables 4 and 5 when $i = 0$ and $r_s = 0.3$.

Place Table 4 here.

Place Table 5 here.

Comparing the policies in Tables 4 and 5 with the optimal policy in Table 3, we see that direct costing over-admits low-revenue arrivals in every period except the last period. In contrast, absorption costing only over-admits in periods $t = 31, 32, 33, 38$ and 39, but under-admits in periods $t = 1, 2, 3, 4$ and 5.

In order to appraise whether the absorption costing policies mix of over- and under-admittance in the nine periods is preferable to direct costing's over-admittance in forty of the forty-one periods, we determine the expected payoffs at $t = 1$ for the two policies. Assume that at the beginning of period 1 the buffers are empty. Then

$$pV_1(0,0,0,2) + (1-p)V_1(0,0,0,1)$$

is a measure of the expected total profit that is generated from the start of the budget period before there are any arrivals. In Table 6, we give this value for each of the three admittance policies.

Place Table 6 here.

The desirability of absorption costing procedures arises because the allocated cost of $K(\mu)/\mu = 0.0952381$ acts as a proxy for the opportunity cost of admitting a low-revenue job. It is, however, clear that if the sunk cost was larger, say $K(\mu) = 3.5$, that this proxy would change. Recalculating the absorption costing admittance policy with all parameters as before except for $K(\mu) = 3.5$ results in absorption costing admitting less jobs into the system because the proxy for opportunity cost now rises to $K(\mu)/\mu = 0.1666667$. The resultant affect on the AC payoff is that now $pV_1(0,0,0,2) + (1-p)V_1(0,0,0,1) = 0.375$. This is less than the payoff obtainable under direct costing. Hence, in general, once the sunk cost becomes sufficiently large, direct costing dominates absorption costing. This arises because the over admittance with direct costing is preferable to the increased under admittance with absorption costing.

We have apparently reached an *impasse* in our attempt to establish the relative desirability of one costing procedure over the other. The example given above illustrates that in general there is no clear dominance. Any differential performance depends on the relative size of the sunk cost allocation. In the next section, we resolve this problem by establishing a bound on the size of the sunk cost that ensures that absorption costing always dominates direct costing.

5 An "Open Admittance" Condition for the Dominance of Absorption Costing

In this section we focus on the admittance of low-revenue orders. If we use the lower bound on the expected time a low-revenue job remains in the system, we know that the DC rule may over-admit but can never under-admit relative to the optimal MDP-based rule. Furthermore, by the nature of the rules, the AC rule may under-admit, but can never over-admit relative to the DC rule. The AC rule can also under- or over-admit relative to the MDP rule. The condition developed in this section ensures that the AC rule used in a given time period *does not under-admit relative to the MDP rule.* This condition, in conjunction with the fact that we are using the lower bounds on the expected holding costs, guarantees that an admittance policy based upon absorption costing generates strictly higher expected profits than a policy based upon direct costing. The intuition for this result is very straightforward. We know that the AC rule never under-admits relative to the DC rule. If, in addition, AC does not under-admit relative to the MDP, then the AC admittance policy must always over-admit less frequently than the DC rule, unless the admittance policies are identical. A graphical representation in Figure 1 makes the point simply.

Place Figure 1 here.

Although the intuition given above is straightforward, the derivation of implementable sufficient conditions is not. The original justification for looking at heuristic cost accounting policies was driven by the difficulty in computing the optimal MDP admittance policy. Hence, j^*_{MDP} is *not observable* and hence cannot form part of the *implementable sufficient conditions.* In what follows, we show how we can develop sufficient conditions in which we replace the unobservable j^*_{MDP} with J, the *observable* maximum buffer size. We can validly do this provided a fairly mild regularity condition (Assumption 1 below) holds.

To simplify the notation in this subsection, let $j^* = j^*_{\text{MDP}}$ denote the optimal admittance rule. This rule depends on n, the number of periods remaining in the budget period, i, the number of high-revenue jobs in the system, and r_s, the revenue of the job currently in service. Since these variables remain fixed throughout our discussion in this section, we do

not explicitly show this dependence. From (8) and (9), we see that

$$A(n, j, r_0) = D(n, j, r_0) - \frac{K(\mu)}{\mu} \sum_{m=j+1}^{n} \mathrm{P}\{N_{j+1} = m\}.$$

Therefore, if

$$D(n, j^* + 1, r_0) > \frac{K(\mu)}{\mu} \sum_{m=j^*+2}^{n} \mathrm{P}\{N_{j^*+2} = m\}, \tag{10}$$

for some n, it follows that

$$A(n, j^* + 1, r_0) > 0. \tag{11}$$

The relationship in (11) has the following interpretation: The AC rule will admit a low-revenue job (if there is room), but it is not optimal to do so (i.e., it over-admits relative to the MDP). (By construction, if there are already $j^* + 1$ low-revenue jobs in the system, the MDP rule will not admit any more.) Rewriting (10) yields,

$$\sum_{m=j^*+2}^{n} \left(r_0 + \frac{n-m}{\Lambda} - \frac{K(\mu)}{\mu} \right) \mathrm{P}\{N_{j^*+2} = m\} > \frac{n}{\Lambda}. \tag{12}$$

Unfortunately, (12) depends upon j^*, a quantity that is not observable. We can establish a condition that does not depend upon j^* by assuming that the following condition holds:

Assumption 1. For some $n \leq N(\lambda, \mu)$,

$$\sum_{m=j}^{n} m\mathrm{P}\{N_j = m\} \tag{13}$$

is increasing in j.

This assumption is fairly weak. Note, for example, that if n is large, (13) is close to $(j+1)/\Lambda$, the mean of N_{j+1}, which is increasing in j. Roughly speaking, the condition requires that n, the number of decision periods remaining in the budget period, must be large relative to j, the number of low-revenue jobs in the system that must be processed. We can show that if (13), the partial expectation of the number of periods that a low-revenue job is in the system, is increasing in j for some $n = n'$, then it is increasing in j for all $n > n'$. It is also straightforward to show that Assumption 1 implies that

$$\sum_{m=j}^{n} \left(\frac{n-m}{\Lambda} \right) \mathrm{P}\{N_j = m\} < \sum_{m=J}^{n} \left(\frac{n-m}{\Lambda} \right) \mathrm{P}\{N_J = m\}, \tag{14}$$

for $j < J$ and $n > J$, where J is the maximum number of low-revenue jobs that can be in the queue.

We now use Assumption 1 and its consequences to simplify the inequality given in (12). The left-side of (12) can be written as

$$\begin{aligned}
&\sum_{m=j^*+2}^{n} \left(r_0 - \frac{K(\mu)}{\mu} + \frac{n}{\Lambda}\right) \mathrm{P}\{N_{j^*+2} = m\} - \frac{1}{\Lambda} \sum_{m=j^*+2}^{n} m\mathrm{P}\{N_{j^*+2} = m\} \\
&\geq \left(r_0 - \frac{K(\mu)}{\mu} + \frac{n}{\Lambda}\right) \mathrm{P}\{N_{j^*+2} \leq n\} - \frac{1}{\Lambda} \sum_{m=J}^{n} m\mathrm{P}\{N_J = m\} \\
&\geq \left(r_0 - \frac{K(\mu)}{\mu} + \frac{n}{\Lambda}\right) \mathrm{P}\{N_J \leq n\} - \frac{1}{\Lambda} \sum_{m=J}^{n} m\mathrm{P}\{N_J = m\} \\
&= \sum_{m=J}^{n} \left(r_0 - \frac{K(\mu)}{\mu} + \frac{n-m}{\Lambda}\right) \mathrm{P}\{N_J = m\}. \qquad (15)
\end{aligned}$$

The first inequality follows from Assumption 1. The second inequality follows from the observation that $\mathrm{P}\{N_{j^*+2} \leq n\} \geq \mathrm{P}\{N_J \leq n\}$. (The left-side of the inequality is the probability that $j^* + 2$ low-revenue jobs can be completed in n or fewer periods, while the right-side is the probability that $J \geq j^* + 2$ jobs can be completed in n or fewer periods.)

We have thus derived a relationship dealing only with the parameters of the problem that guarantees that (10) holds:

Open Admittance Condition

$$\sum_{m=J}^{n} \left(r_0 + \frac{n-m}{\Lambda} - \frac{K(\mu)}{\mu}\right) \mathrm{P}\{N_J = m\} > \frac{n}{\Lambda}. \qquad (16)$$

We call (16) the "Open Admittance" Condition for the following reason: if (16) holds, the AC rule will always admit *any arriving order, regardless of its revenue class*, if there is room in the appropriate input buffer for that job. To see this, note that (16) can be written as $A(n, J, r_0) > 0$. Indirectly, we have shown that if $A(n, j+1, r_0) > 0$, then $A(n, j, r_0) > 0$ for $0 \leq j \leq J - 1$. Therefore, the AC rule will admit a low-reward job no matter how many low-reward jobs are awaiting processing. Furthermore, if the rule always admits low-revenue orders, it will also admit any high-revenue order, if there is room in the input buffer. (To see that (16) implies (10) and, therefore, (11) are true, note that the left-side of (16) is no

larger than the left-side of (12), so that if (16) is true, (12), and hence (10), also hold.) The condition in (16) depends only on observable parameters of the problem and is therefore readily computable.

The intuition behind this condition is as follows. For absorption costing to dominate direct costing, we require that the AC rule admit less low-revenue orders than the DC rule *and* that the AC rule admit a low-revenue order in any situation when the optimal rule does so. That is, we require that the AC rule's proxy for opportunity cost be positive, *but underestimate the true opportunity cost* implicitly used in the MDP. This is what equation (12) formalizes. Note that the requirement that the AC rule never under-admit relative to the optimal admittance policy alleviates the need to compare the losses due to under-admittance with those resulting from over-admittance. The monotonicity requirement given in Assumption 1 allows us to replace the unobservable j^* with J in equation (12), so that the condition is expressed solely in terms of variables that are observable.

The expression given in (16) places restrictions on the relationship between the maximum number of low-revenue jobs that are permitted in the system and the number of time periods that are available for the processing of those jobs. Roughly speaking, the condition states that if the *design* capacity of the system, measured in terms of the size of the input buffer J, is small relative to the number of decision periods remaining in the planning horizon, then absorption costing results in an admittance policy that is closer to the optimal policy than is the direct costing rule. Thus, (16) also formalizes the notion that absorption costing is desirable if allocated costs are not too "too high".

We summarize the primary result developed in this section:

> If the Open Admittance Condition (16) holds, the admittance of low-revenue orders based on absorption costing yields higher expected profits than a rule based on direct costing for all in-process inventory levels.

5.1 Another Numerical Example

The parameters for the example in this section are: $\lambda = 30$, $\mu = 31$, $I = J = 7$, $r_0 = 0.3$, $r_1 = 0.4$, $\beta = 0.5$, $\alpha = 0.2$, $p = 0.2$, and $K(\mu) = 2$. When $n = 41$, Condition 1 and (16) both hold. Therefore, we know that the admittance rule based upon absorption costing dominates

the rule based upon direct costing in the first period of the decision process. Indeed, in the 258 ($8 \times 8 \times 2 \times 2 + 2$) possible states of the process in the first period, the direct costing rule over-admits 76 orders, while the absorption costing rule only over-admits 44 orders. As expected, neither rule ever under-admits an order. Furthermore, the expected profit generated by the absorption costing rule is higher in every state than the profit generated by the direct costing rule. The three admittance policies when $i = 0$ and $r_s = 0.4$ are given in Table 7. The expected payoffs in the initial state are given in Table 8.

Place Table 7 here.

Place Table 8 here.

5.2 Empirical Implications

A number of studies have documented the differential use of either absorption or direct costing practices. For instance, Fremgen and Liao (1981) report that 84% of surveyed companies in the U.S. used absorption costing. Similarly, Atkinson (1987) reports the figure to be 70% for Canadian data. Given this divergence of use, we suggest that empirically-testable hypotheses can be developed from our theory that could explain why some firms use absorption costing while other use direct costing.

It is important to note at this stage that our theory of cost allocation refers to the sunk costs of providing capacity. There are other types of allocated sunk cost, such as corporate headquarter's costs, which are not covered by our theory. A rationale for allocating these other costs may well exist but may be of a different nature. Atkinson (1987, p. 55) argues that besides "... demand for cost allocations ... for the purpose of coordinating the decision making ..." as in our theory, there is also "... other demands for cost allocations which reflect other objectives to be served by cost allocation (particularly the fairness perspective that is prevalent in the regulation, social psychology, taxation, and game theory literatures)." Thus, a general theory of cost allocation would permit different categories of costs that may be allocated for contrasting reasons. An empirical test should clearly differentiate between different categories of cost allocation.

The development of empirical tests of our theory must focus on the allocation of sunk *capacity* costs and not other sunk costs. The satisfaction of our open admittance condition

implies that a manufacturer chooses to admit all arriving orders so that input buffers remain full. One could empirically test to see if the open admittance condition is satisfied by observing a manufacturer's admittance policy. If the manufacturer chooses to fill buffers to capacity, then our theory suggests that this firm is using an admittance rule that is consistent with absorption costing. An alternative to this micro-level analysis of individual manufacturers is to determine those industries that operate at nearly full capacity and test for the usage of absorption costing.

6 Conclusion

There has been considerable debate over whether absorption costing or alternatively direct costing procedures should be used to appraise product order acceptance decisions. The traditional defense of absorption costing is based on its ability to proxy hard to observe opportunity costs. We have established that this argument should not be proposed unless one can demonstrate that the absorption cost based proxies for opportunity costs are indeed "good". We illustrated that in some circumstances absorption costing so overestimates opportunity costs that direct costing is preferable.

We presented a Markov decision process that determines opportunity costs exactly. Since such procedures are difficult to apply in practice, we investigated the extent to which absorption costs are good proxies for opportunity costs. In doing so, we presented one explanation for why the debate over absorption versus direct costing is controversial: in general, neither admittance rule uniformly dominates the other in a stochastic setting.

Further, we contributed to the debate by developing an "open admittance" condition that ensures that an absorption costing admittance rule always generates higher expected profits than does a direct costing rule. Hence, we provided a formal defense of absorption costing procedures when the sufficient conditions we characterize are met. These conditions are couched exclusively in terms of observable variables and hence can be applied in practice. Finally, we briefly discussed the empirical implications of our analysis.

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Parameter	Value	Parameter	Value
λ	20	r_1	0.3
μ	21	β	0.5
I and J	7	α	0.0008
r_0	0.25	p	0.5

Table 1: Parameters values for the numerical example.

(i, j, r_s)	$U_1(i, j, r_s)$	Opportunity Cost $(U_1(i, j+1, r_s) - U_1(i, j, r_s))$
$(0, 0, 2)$	0.53	
$(0, 1, 2)$	0.82	0.29
$(0, 2, 2)$	0.97	0.15
$(0, 3, 2)$	1.02	0.05
$(0, 4, 2)$	1.03	0.01
$(0, 5, 2)$	0.98	-0.05
$(0, 6, 2)$	0.88	-0.10
$(0, 7, 2)$	0.74	-0.14

Table 2: Optimal expected payoffs when $t = 1$.

Time Period t	j^*_{MDP}
1 – 5	4
6 – 30	3
31 – 37	2
38 – 40	1
41 (final period)	0

Table 3: The optimal admittance policy when $i = 0$ and $r_s = 0.3$.

Time Period t	j^*_{DC}
1 – 27	5
28 – 33	4
34 – 37	3
38 – 39	2
40	1
41	0

Table 4: Direct costing admittance rule when $i = 0$ and $r_s = 0.3$.

Time Period t	j^*_{AC}
1 – 33	3
34 – 39	2
40	1
41	0

Table 5: Absorption costing admittance rule when $i = 0$ and $r_s = 0.3$.

Rule	$V_1(0,0,0,2)$	$V_1(0,0,0,1)$	$pV_1(0,0,0,2) + (1-p)V_1(0,0,0,1)$
MDP	0.5283	0.4783	0.5033
DC	0.4900	0.4400	0.4650
AC	0.5277	0.4777	0.5027

Table 6: Expected profit by admittance rule.

Time Period t	j^*_{DC}	j^*_{AC}	j^*_{MDP}
1 – 27	6	6	5
28 – 42	6	6	4
43 – 47	6	5	3
48	5	5	3
49 – 50	5	4	3
51 – 52	4	4	2
53	4	3	2
54 – 55	3	3	2
56	3	2	1
57 – 58	2	2	1
59	1	1	0
60 – 61	0	0	0

Table 7: The three admittance rules when $i = 0$ and $r_s = 0.4$.

Rule	$pV_1(0,0,0,2) + (1-p)V_1(0,0,0,1)$
MDP	0.92348
DC	0.89808
AC	0.90276

Table 8: Expected profit by admittance rule.

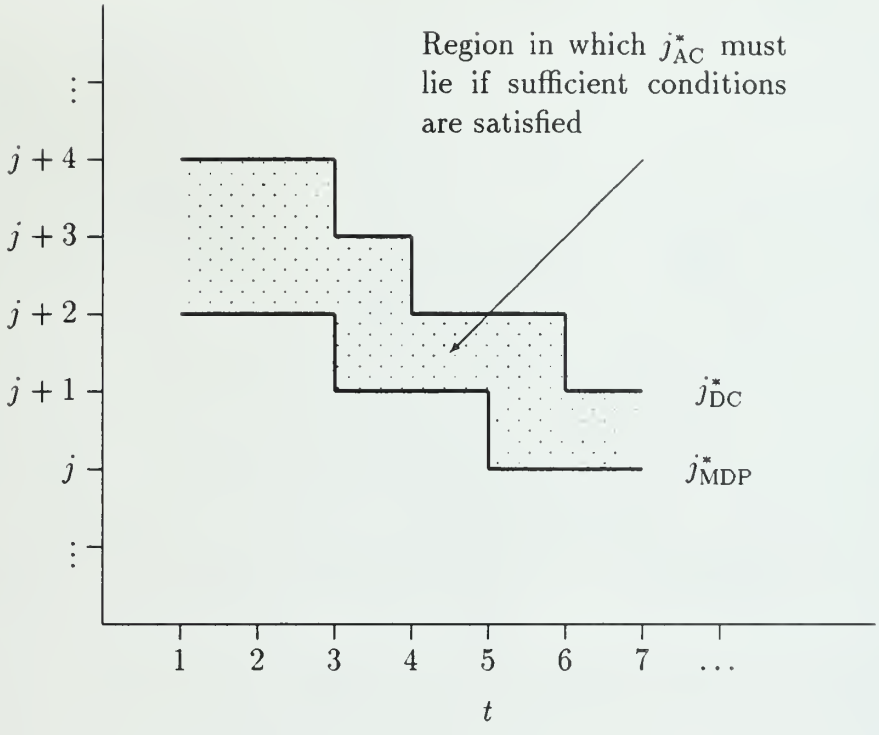

Figure 1: Comparison of Admittance Rules

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